i to the power of i

10.09.2025

Question

1

The imaginary unit $i = \sqrt{-1}$ is difficult to imagine. Because that would mean (if we square both sides):

$$i^2 = -1$$

How can the square of a number ever be negative? Every number multiplied by itself is always positive. But never negative!

Now the innovative mathematician comes along and says, "Yes, it is possible. Because we simply define a special number — we call it "i" — and for this number, its square is exactly minus one."

So let's simply define:

Definition $$i^2 := -1$$

This definition, although difficult to "understand," turns out to be extremely practical: In both electrical engineering and physics, many things can be calculated much more elegantly and expressed more concisely. The imaginary unit i may be strange, difficult to understand, and somewhat mysterious, but it works well when used in calculations.

Now the question …

Now a curious person comes along, looks at the i, thinks about it … and asks the question:

"Then what is i to the power of i?"

and then he even reiterates his question:

"If I raise the imaginary unit i to its own power, what do I get?"

$$i^i = ?$$

Hmm ... well, that's blasphemy! Absolutely forbidden! i itself is completely imaginary. If I raise this number to its own power, what could result?

A kind of "super-imaginary number" that instantly distorts space, burns out brain cells, and creates a bunch of mini-black holes into which we all disappear?

Or is "i to the power of i" a magical spell that reverses causality, inverts laws, and instantly transports us to infinite nirvana?

Idea

2

To solve the mystery, let's simply calculate i^i . And there's a bold approach for this:

We perform the operation "e raised to the power of natural logarithm."

$e^{\ln}$

We can always apply a function followed by its inverse function without anything changing.

Solution

3

$i^i \;=\; e^{\ln i^i} \;=\; e^{i \ln i} \;=\; e^{i\,(i\,\pi/2)} \;=$

$\ln i \;=\; i\;\pi/2$

$i^2 \;=\; -\,1$

$e^{-\,\pi/2} \;=$

But this is a purely real number that can be calculated on any calculator (or by series expansion).

$0,20787.. \sim$

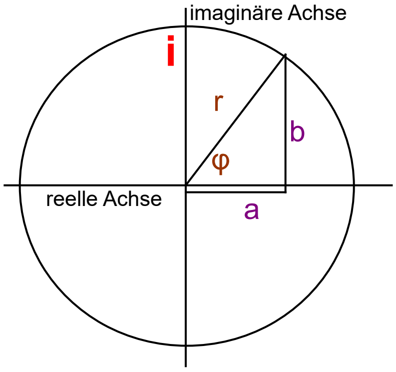
$0,208$

So "i to the power of I" is – quite simply – about one fifth.

General Solution

4

If we look at (ln i), we see that this expression does not have just one value (i π /2), but several values.



$$\ln i = i \frac{\pi}{2} = i\, 5 \frac{\pi}{2} = i\, 9 \frac{\pi}{2} = i\, 13 \frac{\pi}{2} = \ldots$$

More generally we can write:

$$\ln i = i\,(4k + 1) \frac{\pi}{2}$$

$k = 0, 1, 2, \ldots$

The logarithm has an infinite number of values. We then insert these values into the formula:

$$i^i = e^{i \ln i}$$

And we see: The value for "i to the power of i" consists of many, even infinitely many, values:

$$i^i = e^{-(4k+1)\pi/2}$$

$k = 0, 1, 2, \ldots$

As k increases, the value for "i to the power of i" becomes smaller and smaller. It always remains purely real. And it approaches the zero line (or the x-axis) ever closer.

Conclusion

The expression "i to the power of i" denotes infinitely many points that asymptotically approach the zero point starting at 0.208.